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Streaming instability in quantum dusty plasmas

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Abstract. By using a quantum hydrodynamic (QHD) model, we derive a generalized dielectric constant for an unmagnetized quantum dusty plasma composed of electrons, ions, and charged dust particulates. Neglecting the electron inertial force in comparison with the electron pressure, and the force associated with the electron correlations at a quantum scale, we discuss two classes of electrostatic instabilities that are produced by streaming ions, and dust grains. The effects of the plasma streaming speeds, the thermal speed of electrons, and the quantum parameter are examined on the growth rates. The relevance of our investigation to dense astrophysical plasmas is discussed.

PACS. 52.35.-g Waves, oscillations, and instabilities in plasmas and intense beams – 52.30.-q Plasma dynamics and flow – 52.35.Fp Electrostatic waves and oscillations (e.g., ion-acoustic waves) – 52.27.Lw Dusty or complex plasmas; plasma crystals

1 Introduction

Dusty plasmas are composed of electrons, ions and negatively charged dust particulates, have wide ranging applications in space and in industry [1]. The dust particulates are present in different environments of low-temperature laboratory, space, and astrophysical plasmas, i.e. in plasma processing, in plasma coating, in radio-frequency discharges, in tokamak edges, in interstellar media, in interplanetary spaces, in interstellar or molecular clouds, in cometary tails, and in the planetary ring systems. Dust particles are found to be charged either negatively or positively, depending upon different charging mechanisms, viz. the electron-ion sticking on the dust particulates surface from the background plasma, secondary electrons emission, ultra-violet radiations, and thermionic emission, etc. Dusty plasmas support a great variety of wave modes [2–7]. The latter are found to be stable in the equilibrium plasma and become unstable in non-equilibrium plasmas due to an availability of free energy sources [7,8]. The amplitudes of the collective modes grow or damp exponentially with respect to time, depending on the signature of the imaginary wave frequency. If the signature is positive, the waves will grow, and otherwise damp. An upto-date knowledge of the dusty collective modes and associated instabilities are contained in reference [9].

A noticeable interest has been developed for quantum plasma from the beginning of this century. The main reason of attraction is the new development in the manufacturing electronics and other disciplines of physics. For instance, the major role of quantum effects in micro- and nano-electronic devices [10], in dense astrophysical systems [11], and in laser produced plasmas [12] has been recognized. The electron gas in the ordinary metals is also a true quantum plasma. There are two well-known models for describing the quantum mechanical effects in a plasma. The Wigner and Hartree models are based upon the Wigner-Poisson and Schrödinger-Poisson systems, respectively, and present the statistical and hydrodynamic behavior of the plasma particles. The traditional kinetic and fluid models of the plasmas can be recovered by neglecting the quantum effects. The quantum hydrodynamic (QHD) model, which basically deals with the transport of charge, momentum and energy in a plasma, has been introduced for the semiconductor physics to overcome the issues related to resonant tunnelling phenomena, and the negative differential resistance [13]. The investigations [14–21] like drift waves, surface waves, plasma echoes, Landau damping, Zakharov equations, Bernstein-Greene-Kruskal equilibria, and the Debye screening, have

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been studied both analytically and numerically with quantum corrections.

Haas et al. [22] have introduced a quantum multistream model by using the non-linear Schrödinger-Poisson system. They derived the dispersion relations for one- and two-stream plasma instabilities and reported that the two-stream instability is enhanced for small quantum effects in a plasma. Anderson et al. [23] did the same work by employing the Wigner-Poisson system for the purpose to study the statistical effects in the multistream quantum plasma and showed that a Landau-like damping suppresses the instabilities in one- and two-stream type. The stability of small amplitude electrostatic waves has been analyzed by Haas et al. [24] in a three-stream quantum plasma. Lateron, the linear and non-linear properties of the ion acoustic waves have been investigated [25] in a quantum plasma by using the one-dimensional QHD model, and assuming inertialess electrons and mobile ions. Quite recently, a three-dimensional QHD model has been presented by Haas [26] for magnetized plasmas, and the conditions for equilibrium have been established in an ideal quantum magnetohydrodynamics.

It is well-known [27] that in quantum plasmas, the de Broglie wavelength of the charge carriers is comparable to the dimension of the plasma system. In such a situation, the plasma behaves like a Fermi gas, and the quantum mechanical effects are expected to play a crucial role in the behavior of charged plasma particles. For $n_j\lambda_{Bj}^3 \geq 1$, the quantum effect becomes important, where the de Broglie wavelength λ_{Bj} is equal to or greater than the average interparticle distance $d = n_j^{-1/3}$, and n_j is the number density of the jth species (j equals e for electrons, i for ions, and d for dust particulates). On the other hand, for an opposite condition, i.e. $n_j\lambda_{Bj}^3 < 1$, the plasma particles are assumed to behave classically. Recently, the dispersion properties of the linear [28] and non-linear [29] dust acoustic waves have also been studied in an unmagnetized quantum dusty plasma.

In this paper, we calculate a generalized dielectric response function for an unmagnetized quantum dusty plasma whose constituents are the electrons, ions and negatively charged dust particulates. We treat the electrons quantum mechanically and as inertialess, while the ions and the dust particulates are assumed to behave classically. The instability due to the streaming of ions and dust grains are presented, both theoretically and numerically.

The manuscript is organized in the following fashion: in Section 2, we derive a generalized dielectric constant by employing the hydrodynamic and Poisson equations in a quantum dusty plasma. Two specific cases for the instability due to ion-streaming and dust-streaming are investigated. Section 3 summarizes the numerical results and contains the conclusions.

2 Governing equations

We consider a three component quantum dusty plasma consisting of electrons, singly ionized charged ions, and negatively charged dust particulates. The charge-neutrality condition at equilibrium is $\sum_j q_j n_{j0} = 0$, where q_j is the charge ($q_e = -e$ for the electrons, $q_i = e$ for ions, and $q_d = -Z_{d0}e$ for negatively dust particulates, Z_{d0} is the number of the electrons residing on the dust particulate, and e is the magnitude of the electronic charge), and n_{j0} is the equilibrium number density. We look for electrostatic perturbations and suppose that there is no external magnetic field. The dielectric response function in a quantum plasma is governed by the QHD model [18, 25] with streaming of plasma species: the linearized equation of continuity is

$$\frac{\partial n_{j1}}{\partial t} + n_{j0}\frac{\partial U_{j1}}{\partial x} + U_{j0}\frac{\partial n_{j1}}{\partial x} = 0, \quad (1)$$

the linearized equation of motion is

$$\left(\frac{\partial U_{j1}}{\partial t} + U_{j0}\frac{\partial U_{j1}}{\partial x}\right) = -\frac{q_j}{m_j}\frac{\partial \varphi_1}{\partial x} - \frac{1}{m_j n_{j0}}\frac{\partial P_j}{\partial x} + \frac{\hbar^2}{4m_j^2 n_{j0}}\frac{\partial^3 n_{j1}}{\partial x^3}, \quad (2)$$

and the Poisson equation is

$$\frac{\partial^2 \varphi_1}{\partial x^2} = -4\pi \sum_j q_j n_{j1}, \quad (3)$$

where U_{j1} (U_{j0}) is the perturbed (unperturbed) fluid speed, n_{j1} is the perturbed number density, m_j is the mass, φ_1 is the electrostatic potential, and $P_j = n_{j1}k_B T_j$ is the pressure representing the isothermal equation of state. T_j is the temperature, k_B is the Boltzmann constant, and $\hbar$ is the Plank constant divided by 2π. The present investigation is relevant to dense astrophysical plasma [11] even though it is hot but still exists in quantum state. The quantum statistical effects are not included and play a role in the study of ordinary metals, metal clusters, and nanoparticles, where the electron Fermi temperature is much higher than the room temperature. Equations (1)–(3) describe the dynamics of the quantum plasma and are obtained by solving the nonlinear Schrödinger-Poisson system [22]. The third term in the right-hand side of equation (2) represents the quantum diffraction effects and its origin is due to quantum correlation of density fluctuations [27]. We assume a plane wave solution of the form $\exp(ikx - i\omega t)$ for the perturbed quantities, where k (ω) is the wavenumber (angular wave frequency) of propagating waves. Calculating n_{j1} and U_{j1} from equations (1) and (2), respectively, we obtain

$$n_{j1} = \frac{k n_{j0}}{(\omega - kU_{j0})}U_{j1}, \quad (4)$$

and

$$U_{j1} = \frac{1}{(\omega - kU_{j0})}\left[\frac{q_j}{m_j}k\varphi_1 + \frac{k_B T_j}{m_j n_{j0}}k n_{j1} + \frac{\hbar^2}{4m_j^2 n_{j0}}k^3 n_{j1}\right]. \quad (5)$$

Substituting equation (5) into equation (4), we have

$$n_{j1} = \frac{k n_{j0}}{(\omega - kU_{j0})^2}\left[\frac{q_j}{m_j}k\varphi_1 + \frac{k_B T_j}{m_j n_{j0}} k n_{j1} + \frac{\hbar^2}{4m_j^2 n_{j0}} k^3 n_{j1}\right], \quad (6)$$

which can be further simplified as

$$n_{j1} = \frac{q_j n_{j0}}{m_j}\left[\frac{k^2\varphi_1}{(\omega - kU_{j0})^2 - k^2V_{tj}^2 - \hbar^2k^4/4m_j^2}\right]. \quad (7)$$

From equations (3) and (7), we then obtain

$$1 - \sum_{j=e,i,d} \frac{\omega_{pj}^2}{(\omega - kU_{j0})^2 - k^2V_{tj}^2 - \hbar^2k^4/4m_j^2} = 0, \quad (8)$$

which is the generalized form of the dielectric response function in a quantum dusty plasma. Here, $\omega_{pj} = \sqrt{4\pi q_j^2 n_{j0}/m_j}$ is the plasma frequency, and $V_{tj} = \sqrt{k_B T_j/m_j}$ is the thermal speed. Equation (8) can be written in the following form

$$1 - \frac{\omega_{pe}^2}{(\omega - kU_{e0})^2 - \Omega_e^2} - \frac{\omega_{pi}^2}{(\omega - kU_{i0})^2 - \Omega_i^2} - \frac{\omega_{pd}^2}{(\omega - kU_{d0})^2 - \Omega_d^2} = 0, \quad (9)$$

where $\Omega_e^2 = k^2V_{te}^2 + \hbar^2k^4/4m_e^2$, $\Omega_i^2 = k^2V_{ti}^2 + \hbar^2k^4/4m_i^2$, and $\Omega_d^2 = k^2V_{td}^2 + \hbar^2k^4/4m_d^2$. In the following subsections, we consider two specific cases of instability due to the ion-streaming, and the dust-streaming. We shall ignore the quantum effects for the ions and the dust particulates due to their large mass compared to the electrons.

2.1 Ion-streaming instability

Here, we consider the approximations $|\omega - kU_{e0}| \ll \Omega_e$, $|\omega - kU_{i0}| \gg \Omega_i$, and $\omega \gg kU_{d0}$, Ω_d, and write equation (9) as

$$1 + \frac{\omega_{pe}^2}{k^2V_{te}^2 + \hbar^2k^4/4m_e^2} - \frac{\omega_{pi}^2}{(\omega - kU_{i0})^2} - \frac{\omega_{pd}^2}{\omega^2} = 0, \quad (10)$$

equation (10) can be put in the form

$$1 - \frac{\Omega_{IA}^2}{(\omega - kU_{i0})^2} - \frac{\Omega_{DA}^2}{\omega^2} = 0, \quad (11)$$

where

$$\Omega_{IA} = \omega_{pi}\sqrt{(k^2V_{te}^2 + \hbar^2k^4/4m_e^2)/(k^2V_{te}^2 + \omega_{pe}^2 + \hbar^2k^4/4m_e^2)}$$

and

$$\Omega_{DA} = \omega_{pd}\sqrt{(k^2V_{te}^2 + \hbar^2k^4/4m_e^2)/(k^2V_{te}^2 + \omega_{pe}^2 + \hbar^2k^4/4m_e^2)}$$

are the ion-acoustic and dust-acoustic frequencies, respectively. Letting $\omega = kU_{i0} + \delta$ in equation (11), assuming $\delta < kU_{i0}$, and $kU_{i0} \sim \Omega_{DA}$, we obtain

$$\delta^3 = \frac{1}{2}\Omega_{IA}^2 kU_{i0}. \quad (12)$$

The roots of equation (12) are

$$\delta = \left(\frac{\Omega_{IA}^2 kU_{i0}}{2}\right)^{1/3}, \ \left(\frac{-1 + i\sqrt{3}}{2}\right)\left(\frac{\Omega_{IA}^2 kU_{i0}}{2}\right)^{1/3}, \ \left(\frac{-1 - i\sqrt{3}}{2}\right)\left(\frac{\Omega_{IA}^2 kU_{i0}}{2}\right)^{1/3}. \quad (13)$$

We express the second root of δ in terms of real and imaginary parts, respectively, as

$$\tilde{\omega}_r = -\frac{1}{2^{4/3}}\left[\frac{\tilde{K}\,\tilde{\omega}_{pi}^2(\tilde{K}^2\tilde{V}_{te}^2 + \tilde{K}^4H^2/4)}{(\tilde{K}^2\tilde{V}_{te}^2 + \tilde{\omega}_{pe}^2 + \tilde{K}^4H^2/4)}\right]^{1/3}, \quad (14)$$

and

$$\tilde{\omega}_i = \frac{\sqrt{3}}{2^{4/3}}\left[\frac{\tilde{K}\,\tilde{\omega}_{pi}^2(\tilde{K}^2\tilde{V}_{te}^2 + \tilde{K}^4H^2/4)}{(\tilde{K}^2\tilde{V}_{te}^2 + \tilde{\omega}_{pe}^2 + \tilde{K}^4H^2/4)}\right]^{1/3}. \quad (15)$$

Here, equations (14) and (15) are normalized by the parameters: $\tilde{\omega}_{r,i} = \omega_{r,i}/\omega_{pi}$, $\tilde{\omega}_{pi} = 1$, $\tilde{\omega}_{pe} = \omega_{pe}/\omega_{pi}$, $\tilde{K} = kU_{i0}/\omega_{pi}$, $\tilde{V}_{te} = V_{te}/U_{i0}$, and $H = \hbar\omega_{pi}/m_eU_{i0}^2$ is the quantum parameter representing the quantum correction due to the density fluctuations.

2.2 Dust-streaming instability

Here, we suppose that the dust grains are streaming against the electrons and ions. Introducing the approximations $|\omega - kU_{e0}| \ll \Omega_e$ and $\omega \gg kU_{i0}$, Ω_i, we then obtain from equation (9)

$$1 + \frac{\omega_{pe}^2}{k^2V_{te}^2 + \hbar^2k^4/4m_e^2} - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pd}^2}{(\omega - kU_{d0})^2} = 0, \quad (16)$$

or

$$1 - \frac{\Omega_{IA}^2}{\omega^2} - \frac{\Omega_{DA}^2}{(\omega - kU_{d0})^2} = 0. \quad (17)$$

Letting $\omega = kU_{d0} + \Delta$ in equation (17) where $\Delta < kU_{d0}$, and $kU_{d0} \sim \Omega_{IA}$, we obtain

$$\Delta^3 = \frac{1}{2}\Omega_{DA}^2 kU_{d0}. \quad (18)$$

The roots of equation (18) are

$$\Delta = \left(\frac{\Omega_{DA}^2 kU_{d0}}{2}\right)^{1/3}, \left(\frac{-1 + i\sqrt{3}}{2}\right)\left(\frac{\Omega_{DA}^2 kU_{d0}}{2}\right)^{1/3}, \left(\frac{-1 - i\sqrt{3}}{2}\right)\left(\frac{\Omega_{DA}^2 kU_{d0}}{2}\right)^{1/3}, \quad (19)$$

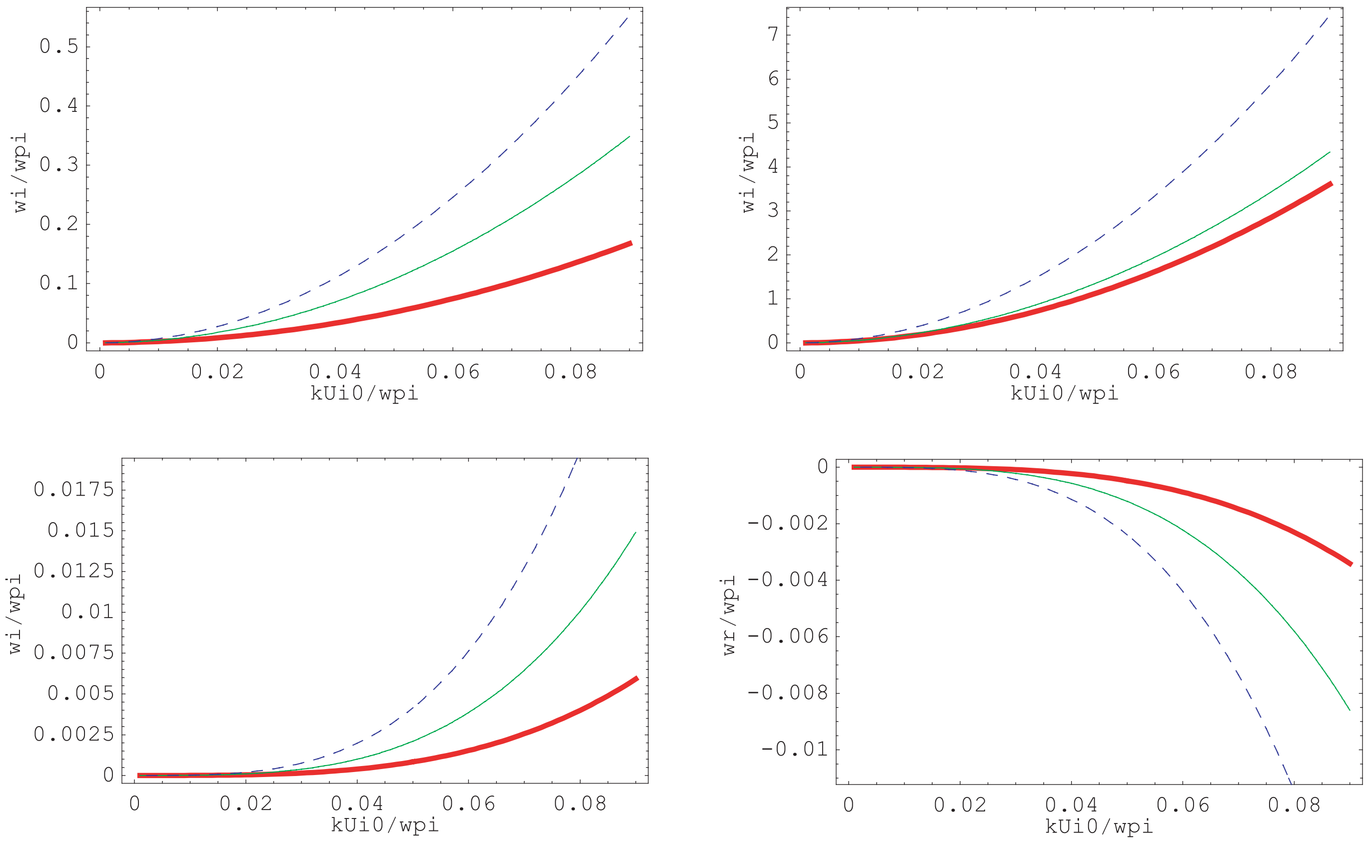

Fig. 1. (Color online) The normalized growth rates and the normalized real angular frequencies ($\tilde{\omega}_{i,r} = \omega_{i,r}/\omega_{pi}$) are plotted against the normalized wavenumber ($\tilde{K} = kU_{i0}/\omega_{pi}$) using equations (14) and (15) for (i) different ion-streaming speeds $U_{i0} = 0.0001\omega_{pi}$, thick curve; $U_{i0} = 0.0003\omega_{pi}$, thin curve; $U_{i0} = 0.0006\omega_{pi}$, dashed curve (top left figure), (ii) a pure classical case ($H = 0$) with changing electron thermal speed $\tilde{V}_{te} = (0.01, 0.02, 0.03)$ and $U_{i0} = 0.01\omega_{pi}$ (top right), (iii) a pure quantum case ($\tilde{V}_{te} = 0$) with varying $H = 0.3, 0.6, 1$ and keeping $U_{i0} = 0.01\omega_{pi}$ (bottom left), and (iv) different values of $H = 0.3, 0.6, 1$ with fixed $\tilde{V}_{te} = 0$ and $U_{i0} = 0.01\omega_{pi}$ (bottom right). Other parameters used in our numerical calculations are: $m_e = 9.1\times10^{-28}$ g, $m_i = 12m_p$ (m_p is the mass of proton), $m_d = 4.008 \times 10^{-16}$ g, $n_{e0} \approx n_{i0} = 1.0 \times 10^{19}$ cm^{-3}, $n_{d0} = 1.0 \times 10^{14}$ cm^{-3}, $Z_{d0} = 10^4$, and $T_e = 10^5$ K.

Introducing the normalizations: $\tilde{\omega}_{r,i} = \omega_{r,i}/\omega_{pd}$, $\tilde{\omega}_{pd} = 1$, $\tilde{\omega}_{pe} = \omega_{pe}/\omega_{pd}$, $\tilde{K} = kU_{d0}/\omega_{pd}$, $\tilde{V}_{te} = V_{te}/U_{d0}$, and $H = \hbar\omega_{pd}/m_e U_{d0}^2$, we obtain for the real and imaginary parts from the second root of equation (19), respectively, as

$$\tilde{\omega}_r = -\frac{1}{2^{4/3}}\left[\frac{\tilde{K}\,\tilde{\omega}_{pd}^2(\tilde{K}^2\tilde{V}_{te}^2 + \tilde{K}^4H^2/4)}{(\tilde{K}^2\tilde{V}_{te}^2 + \tilde{\omega}_{pe}^2 + \tilde{K}^4H^2/4)}\right]^{1/3}, \qquad (20)$$

and

$$\tilde{\omega}_i = \frac{\sqrt{3}}{2^{4/3}}\left[\frac{\tilde{K}\,\tilde{\omega}_{pd}^2(\tilde{K}^2\tilde{V}_{te}^2 + \tilde{K}^4H^2/4)}{(\tilde{K}^2\tilde{V}_{te}^2 + \tilde{\omega}_{pe}^2 + \tilde{K}^4H^2/4)}\right]^{1/3}. \qquad (21)$$

3 Numerical results and conclusions

For the numerical evaluation of the growth rates and real angular frequencies, we choose some typical parameters that are representative of dense astrophysical plasmas [11,18,19]: $m_e = 9.1 \times 10^{-28}$ g, $m_i = 12m_p$ (m_p is the mass of proton), $m_d = 4.008 \times 10^{-16}$ g, $n_{e0} \approx n_{i0} = 1.0 \times 10^{19}$ cm^{-3}, $n_{d0} = 1.0 \times 10^{14}$ cm^{-3}, $T_e = 10^5$ K, and the dust charge state $Z_{d0} = 10^4$. Using these parameters, we solve equations (14), (15), (20) and (21) numerically, and examine the effects of the ion-dust-streaming speeds (U_{i0}, U_{d0}), the thermal speed of the electrons (V_{te}), and the dimensionless quantum parameter (H) on the growth rates. The quantum effects are also observed on the profiles of the real angular frequency. The normalized growth rates ($\tilde{\omega}_i$) and the normalized real angular wave frequencies ($\tilde{\omega}_r$) are plotted against the normalized wavevector $\tilde{K}$ for (i) different normalized ion-streaming speeds $U_{i0} = (0.0001, 0.0003, 0.0006)$, (ii) a pure classical case ($H = 0$) with $U_{i0} = 0.01\,\omega_{pi}$ and $\tilde{V}_{te} = (0.01, 0.02, 0.03)$, (iii) a pure quantum case ($\tilde{V}_{te} = 0$) with changing $H = (0.3, 0.6, 1)$ and keeping $U_{i0} = 0.01\omega_{pi}$, and (iv) choosing different values of $H = 0.3, 0.6, 1$ having $U_{i0} = 0.01\omega_{pi}$ and $\tilde{V}_{te} = 0$. Figure 1 shows that the growth rates as well as the real angular frequencies increase with increasing ion-streaming speeds, the quantum parameter, and the thermal speed of the electrons. Furthermore, Figure 2 displays the variation

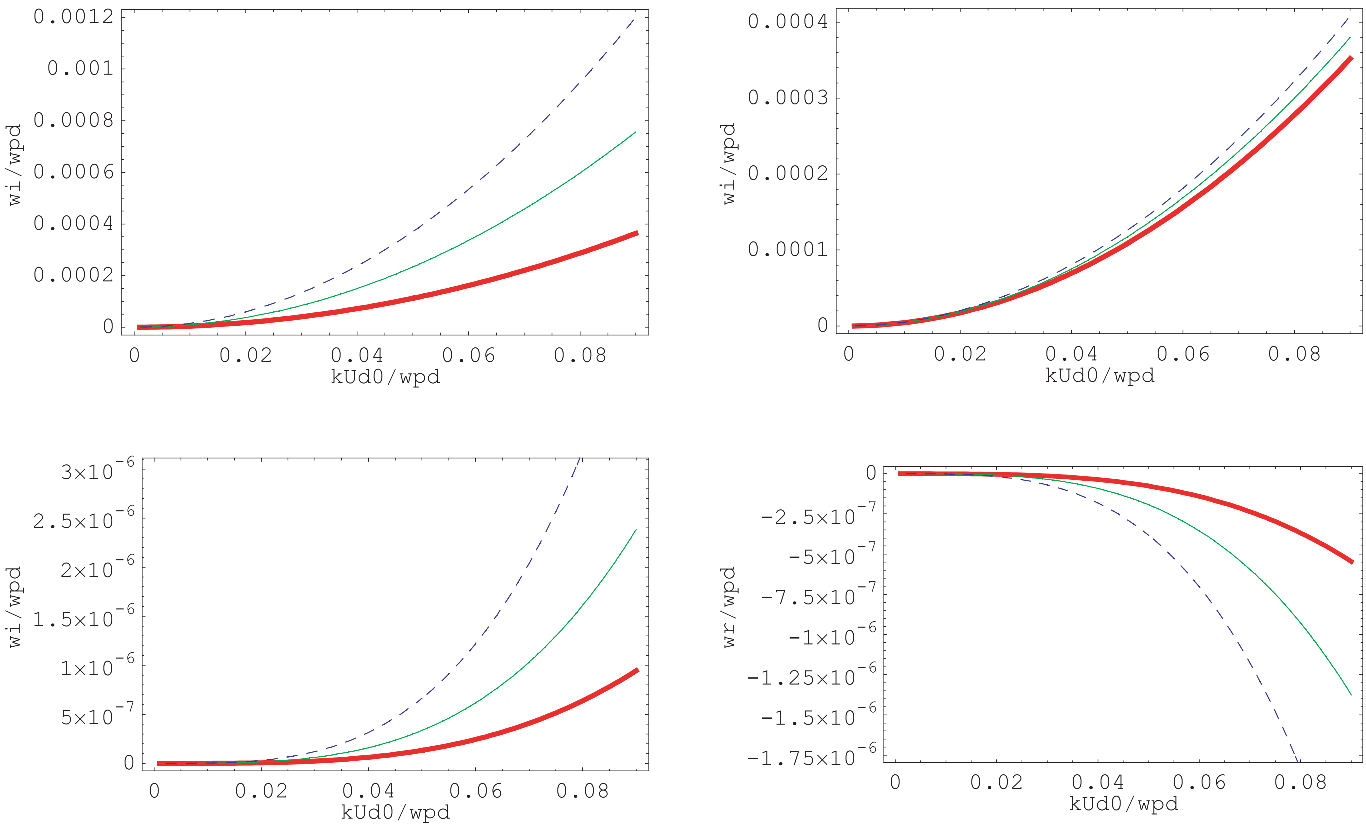

Fig. 2. (Color online) The normalized growth rates and the normalized real angular frequencies ($\tilde{\omega}_{i,r} = \omega_{i,r}/\omega_{pd}$) are plotted against the normalized wavenumber ($\tilde{K} = kU_{d0}/\omega_{pd}$) using equations (20) and (21) for (i) different dust-streaming speeds $U_{d0} = 0.001\omega_{pd}$, thick curve; $U_{d0} = 0.004\omega_{pd}$, thin curve; $U_{d0} = 0.008\omega_{pd}$, dashed curve (top left figure), (ii) a pure classical case ($H = 0$) with different electron thermal speed $\tilde{V}_{te} = (17, 18, 19)$ and $U_{d0} = 0.001\omega_{pd}$ (top right), (iii) a pure quantum case ($\tilde{V}_{te} = 0$) with $H = 0.03, 0.06, 0.1$ and fixed $U_{d0} = 0.14\omega_{pd}$ (bottom left), and (iv) different values of $H = 0.03, 0.06, 0.1$ having $\tilde{V}_{te} = 0$ and $U_{d0} = 0.14\omega_{pd}$ (bottom right). All other parameters are the same as in Figure 1.

of the growth rates and the real angular frequencies due to the dust-streaming speeds in quantum plasmas.

To conclude, we have presented a generalized dielectric response function for the quantum dusty plasma by using the QHD model, and the Poisson equation. In such a plasma, the electron density perturbation is affected by the quantum correction, whereas the positive ions and negatively charged dust particulates behave classically. Two specific cases for the instability due to the ion-streaming and the dust-streaming are presented both analytically and numerically. It is found that the growth rates and the real angular frequencies are significantly affected by the variation of the streaming speeds, the thermal speeds, and the quantum effects. Finally, our results may be useful to understand the underlying physics of dense astrophysical quantum dusty plasmas containing ions and dust flows.

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